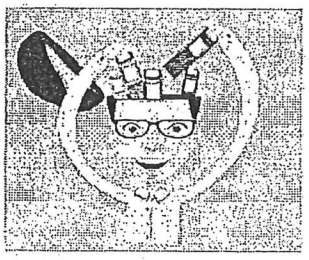
AP Calculus AB

Summer Packet

This packet is intended to prepare you for AP Calculus AB. You will review algebra and pre-calculus skills. It is due on the first day of school in August and will be worth homework points based on completion. The packet is a little lengthy so please do not wait until the last minute to get started. The first page has properties and concepts that will help you complete the packet. The 'library of functions' page should be filled out by you and kept in your "toolbox". The toolbox is a section of the three-ring binder you need for this course. I filled in the first two functions as an example. You should know what these functions look like and how they behave. You will also be required to have a TI 84 calculator. We will work hard in this class, but it will be worth it. Calculus is a fascinating course; I know you will love it as much as I do. Have a wonderful summer!

Mrs. May

The following formulas and identities will help you complete this packet. You are expected to know ALL of these for the course.



LINES

Slope-intercept: $y = mx + b$

Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$

Horizontal line: $y = b$ (slope $= 0$)

Vertical line: $x = a$ (slope = undefined)

Parallel → same slope

Perpendicular → opposite reciprocal slopes

QUADRATICS

Standard: $y = ax^2 + bx + c$

Vertex: $y = a(x - h)^2 + k$

Intercept: $y = a(x - p)(x - q)$

Parabola opens: up if $a > 0$
down if $a < 0$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXPONENTIAL PROPERTIES

$x^a \cdot x^b = x^{a+b}$ $\quad (xy)^a = x^a y^a$

$\frac{x^a}{x^b} = x^{a-b}$ $\quad \sqrt[n]{x^m} = x^{m/n}$

$x^0 = 1\ (x \neq 0)$ $\quad \left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$

$x^{-n} = \frac{1}{x^n}$

In general, it is fine to have negative exponents in your answers!

LOGARITHMS

$y = \log_a x$ is equivalent to $a^y = x$

$\log_b(mn) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \log_b m$

TRIGONOMETRIC IDENTITIES

$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cot x = \frac{1}{\tan x}$ $\quad \tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$

$\sin^2 x + \cos^2 x = 1$ $\quad \tan^2 x + 1 = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

$\sin(2x) = 2 \sin x \cos x$ $\quad \cos(2x) = \cos^2 x - \sin^2 x$ **or** $1 - 2\sin^2 x$ **or** $2\cos^2 x - 1$

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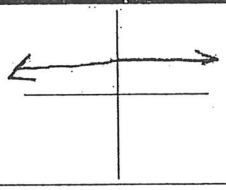
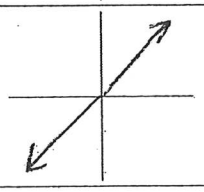
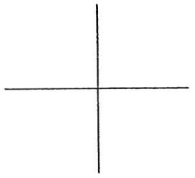
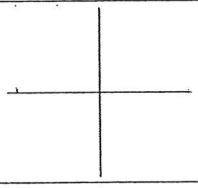
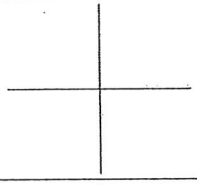
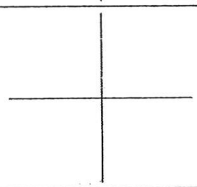
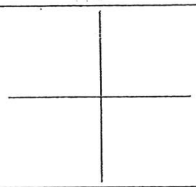
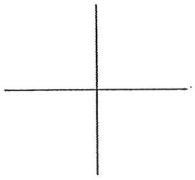
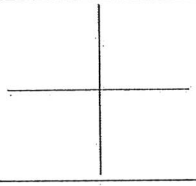
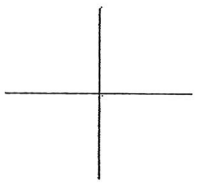
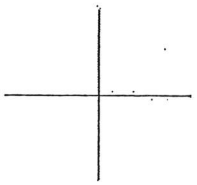
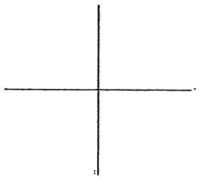
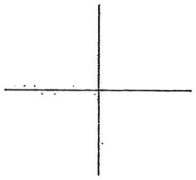
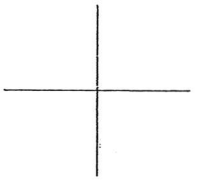
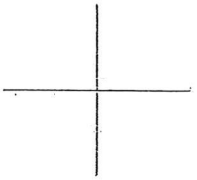
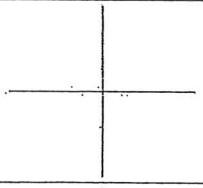
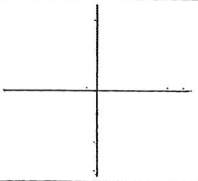
Library of Functions

	Function	Graph	Domain (Set)	Domain (Int)	Range (Set)	Range (Int)
1	Constant $f(x)=a$		$\mathbb{R}$	$(-\infty,\infty)$	$y=a$	$[a]$
2	Identity $f(x)=x$		$\mathbb{R}$	$(-\infty,\infty)$	$\mathbb{R}$	$(-\infty,\infty)$
3	Absolute Value					
4	Quadratic					
5	Square Root					
6	Reciprocal					
7	Cubic					
8	Cube Root					
9	Greatest Integer					

10	Reciprocal of a Square					
11	Semicircle					
12	Exponential Growth					
13	Exponential Decay					
14	Natural Logarithm					
15	Sine					
16	Cosine					
17	Arctangent					

Name: _____

AP Calculus AB Summer Assignment 2023

Simplify.

1. $16^{\frac{3}{4}}$

2. $8^{\frac{2}{3}}$

3. $16^{\frac{3}{2}}$

4. $-27^{\frac{2}{3}}$

5. $(-32)^{\frac{4}{5}}$

6. Write the exponential expression $3x^{\frac{3}{8}}$ in radical form.

7. Write the radical expression $\frac{8}{\sqrt[7]{x^{15}}}$ in exponential form.

For #1-8, write an equation in point-slope form.

8. Containing (4, -1) with a slope of $\frac{1}{2}$

9. Crossing the x-axis at x = -3 and the y-axis at y = 6.

10. Containing the points (-6,-1) and (3, 2).

11. Passing through (5, -3) with an undefined slope.

12. Passing through (-4, 2) with a slope of 0.

13. Passing through (2, 8) and parallel to $y = \frac{5}{6}x - 1$.

14. Passing through (6, -7) and perpendicular to $y = -2x - 5$

For #9-13, solve each equation for x. Note that some will have a particular value but others will have a solution in terms of other variables.

15. $x^2 + 3x = 8x - 6$

16. $\frac{2x-5}{x+y}=3-y$

17. $3xy+6x-xz=12$

18. $A=ax+bx$

19. $\frac{y+2}{4-x}=4(2-z)$

For #14-18, solve each quadratic by factoring.

20. $x^2-4x-12=0$

21. $x^2-6x+9=0$

22. $x^2-36=0$

23. $4x^2+4x+1=0$

For #19-23, evaluate the following knowing that $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$.

24. $f\left(\frac{1}{2}\right)$

25. $g(-2)$

26. $f(1) + g(0)$

27. $f(0) \cdot g(0)$

28. $\frac{g(-6)}{f(-6)}$

For #24-29, use $f(x) = x^2 - 1, g(x) = 3x$ and $h(x) = 5 - x$ to find each composite function.

29. $f(g(x))$

30. $g(f(x))$

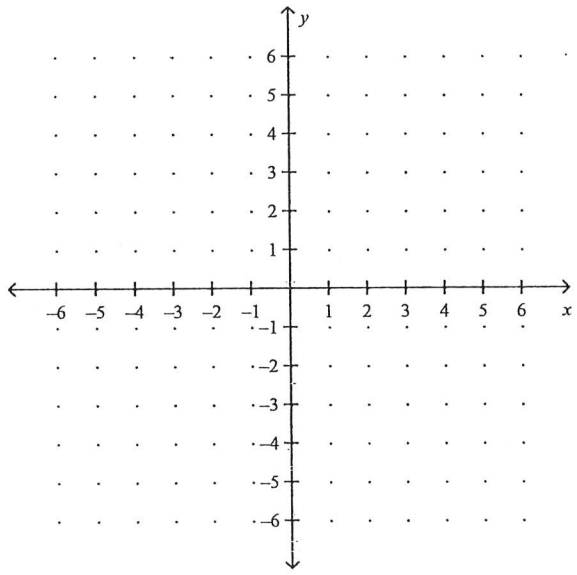
31. $f(f(4))$

32. $g(h(-4))$

33. $f(g(h(1)))$

For #30-31, graph each piecewise function.

34. $f(x) = \begin{cases} x+3; & x < 0 \\ -2x+5; & x \geq 0 \end{cases}$



For #35-38, simplify each expression without the use of a calculator. The exponetial properties on page two of this packet will help.

35. $e^{\ln 4}$

36. $e^{2\ln 3}$

37. $\ln e^9$

38. $5\ln e^3$

For #39-43, solve each exponential or logarithmic equation by hand.

39. $e^x = 34$

40. $3e^x = 120$

41. $e^x - 8 = 51$

42. $\ln x = 2.5$

43. $\ln(3x - 2) = 2.8$

For #44-50, give the exact value of the expression using the unit circle.

44. $\cos\left(\frac{11\pi}{6}\right) =$

45. $\sin\left(\frac{-2\pi}{3}\right) =$

46. $\tan\left(\frac{7\pi}{4}\right) =$

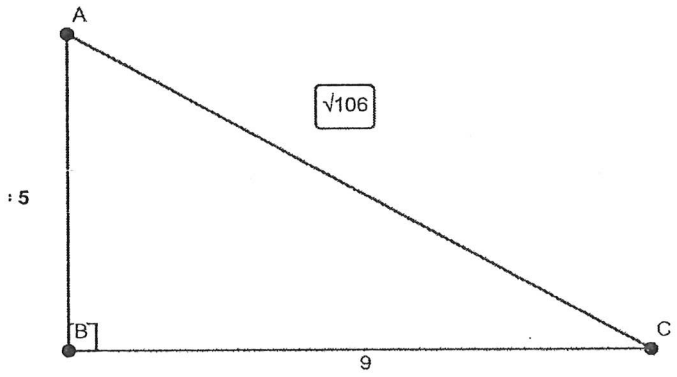
47. $\csc\left(\frac{\pi}{4}\right) =$

48. $\cot\left(\frac{5\pi}{4}\right) =$

49. $\sec\left(\frac{-\pi}{4}\right) =$

50. $\tan\left(\frac{-4\pi}{3}\right) =$

Evaluate each trig function using the right triangle provided. No need to rationalize the denominator.



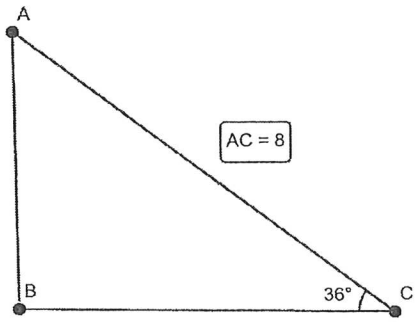
51. $\sin C =$

52. $\cos C =$

53. $\tan A =$

54. $\sec A =$

55. **Solve the triangle, rounding all angles and sides to the nearest thousandth.**



m<A = _____

AB = _____

CB = _____

For #56-60, evaluate each inverse trig function using the unit circle. Write all answers in radians, not degrees. Do not use a calculator.

56. $\sin^{-1}\left(\frac{1}{2}\right) =$

57. $\sin^{-1}(-1) =$

58. $\arccos\left(\frac{-\sqrt{2}}{2}\right) =$

59. $\arctan(-1) =$

60. **Explain how the graph of $f(x)$ and its inverse, $f^{-1}(x)$, compare.**

_____

_____

_____

For #62-64, find the inverse of each function.

61. $g(x) = \dfrac{5}{x-2}$

62. $f(x) = \dfrac{x^2}{3}$

63. $y = \sqrt{4-x} + 1$

64. **Consider the function** $f(x) = \dfrac{e^x}{\log x - x^3}$.

a. Use your calculator to find the relative maximum and minimum of f(x).

Min value = _____ **max value =** _____

b. State the domain of $f(x)$ **in interval notation.**

D: _____

c. State when the function is increasing and decreasing. Write in interval notation.

increasing _____ **decreasing** _____